DERIVATION OF EFFECTIVE WAVE EQUATION FOR VERY-HIGH-FREQUENCY SHORT WAVES IN BUBBLY LIQUIDS

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SUMMARY
The present study theoretically investigates the propagation of plane progressive waves of very high frequency in a quiescent liquid containing many spherical gas bubbles. We focus on a compressibility of liquid phase that has been neglected in many previous studies, and treat wave propagation in a mode, i.e., “Fast mode”, induced by the liquid compressibility. Waves in Fast mode propagate with large phase velocity exceeding the sound speed in liquid. We derive an effective wave equation for high frequency short wave in Fast mode. By using the derivation method for effective wave equations in bubbly liquids recently proposed by our group, a linear equation with dissipation and dispersion effects can be derived from a set of basic equations in a two-fluid model.

INTRODUCTION
Dispersion waves in bubbly liquids
As it is widely known, waves in a liquid containing a large number of small gas bubbles exhibit fairly complex behaviors as compared with those in single phase fluids as a result of complexity of two-phase fluids and variety of wave properties such as the dispersion, dissipation, and interference.

Especially, the most important and fundamental feature in the waves in bubbly liquids is the dispersion, a simple schematic is shown in Fig. 1, where the relation between the frequency and wavenumber of monochromatic progressive wave is plotted in the case that the amplitude of the wave is sufficiently small [1]. There are two branches: the upper branch appears when the compressibility of bubbly liquid is predominant in the wave propagation processes, while the lower branch can be observed in all situations. Note that the dissipation effects are neglected in the dispersion relation depicted in Fig. 1.

On the upper branch, the phase velocity is always larger than the speed of sound in the absence of bubbles (i.e., single phase liquid such as pure water) signified by \( c_{L0}^* \) and on the lower branch, the phase velocity is always smaller than \( c_{L0}^* \); the superscript * denotes the dimensional quantities and the subscript 0 does an initial unperturbed state throughout this paper. Hence, the former may be called Fast mode and the latter Slow mode [1].

Unified derivation method of wave equations
A number of theoretical studies on waves in bubbly liquids have been carried out so far [2, 3, 4, 5, 6], and various types of effective equations for wave propagation, which are sometimes called nonlinear wave equations, have been derived. The most famous nonlinear wave equations are the Korteweg–de Vries–Burgers (KdVB) equation [2, 3] for a long wave with a low frequency and the nonlinear Schrödinger (NLS) equation [5] for an envelope of a carrier wave with a high frequency. The existence of various types of nonlinear wave equations is an evidence of complexity of waves in bubbly liquids. Systematic studies of these complex phenomena are important for the understanding of physical mechanism and the enhancement of their applications.

In our previous paper [7], we therefore have proposed a sys-
The present study therefore focuses on waves in Fast mode. We shall decompose Fast mode into Band A and Band B as illustrated in Fig. 3, where Band A and Band B correspond to a short wave in a very high frequency and a long wave in a moderately high frequency slightly above the cutoff frequency, respectively. By using the unified theory [7] described in the preceding subsection, this paper demonstrates the derivation of an effective wave equation for Band A in Fast mode.

**FORMULATION**

**Problem statement**

We shall examine one-dimensional nonlinear dispersive waves in a bubbly liquid made up of a compressible liquid and a large number of small spherical gas bubbles. At an initial state, the bubbly liquid is assumed to be uniform and at rest. The pressure waves are generated from a sound source placed in the bubbly liquid. The amplitude of pressure wave is sufficiently small compared with the pressure in the ambient bubbly liquid.

Let us here summarize the main assumptions:

(i) The motion of bubble is spherically symmetric.

(ii) The bubbles do not coalesce, break up, extinct, and appear.

(iii) The direct bubble–bubble interaction does not occur.

(iv) The volume fraction of gas phase (void fraction) in the bubbly liquid is uniform at the initial state.

(v) The compressibility of the liquid is taken into account.

(vi) The gas inside bubbles is composed of only a non-condensable gas, and hence the phase change across the bubble–liquid interface does not occur.

(vii) Both the viscosities and the bulk viscosities of gas and liquid are neglected.

(viii) The thermal conductivities of the gas and liquid, Reynolds stress, and gravitation, are dismissed.

In general, the wave attenuation is caused by the three effects [3]: the liquid viscosity, liquid compressibility, and thermal conductivity. It is well known that the thermal process inside bubbles with the heat exchange at the bubble–liquid interface induces a significant attenuation of waves in bubbly liquids (e.g., Watanabe & Prosperetti [10]). In this paper, however, we neglect the thermal effect for simplicity, and we take into account the wave attenuation due to only the liquid compressibility. As in clear from the above assumptions (v)–(viii), we have taken into account the wave attenuation due to only the liquid compressibility.
Basic equations

We shall use a set of basic equations for bubbly flows recently proposed by our group (see, Egashira et al. [11] and Yano et al. [11]), which are composed of conservation equations of mass and momentum for the gas and liquid phases based on the so-called two-fluid model [1], the equation of motion for the bubble wall, the equations of state for the gas and liquid phases, and so on.

For one-dimensional flows, firstly, the conservation equations of mass and momentum are given by

\[
\frac{\partial}{\partial t^*}(\alpha \rho_*^G u_G^*) + \frac{\partial}{\partial x^*}(\alpha \rho_*^G u_G^*) = 0, \tag{2}
\]

\[
\frac{\partial}{\partial t^*}[\alpha \rho_*^L u_G^*] + \frac{\partial}{\partial x^*}[\alpha \rho_*^L u_G^*] = 0, \tag{3}
\]

\[
\frac{\partial}{\partial t^*}(\alpha \rho_*^G u_G^*) + \frac{\partial}{\partial x^*}(\alpha \rho_*^G u_G^*)^2 + \frac{\alpha}{\alpha x^*} \frac{\partial \rho_*^G}{\partial x^*} = F^*, \tag{4}
\]

\[
\frac{\partial}{\partial t^*}(\alpha \rho_*^L u_G^*) + \frac{\partial}{\partial x^*}(\alpha \rho_*^L u_G^*)^2 + \frac{\alpha}{\alpha x^*} \frac{\partial \rho_*^L}{\partial x^*} = -F^*, \tag{5}
\]

where \( t^* \) is the time, \( x^* \) is the space coordinate normal to the wave front, \( \alpha \) is the void fraction \( (0 < \alpha < 1) \), \( \rho^* \) is the density, \( u^* \) is the fluid velocity, \( \rho^* \) is the pressure, and the subscripts G and L denote volume-averaged variables in the gas and liquid phases, respectively. In addition to the volume-averaged pressures \( \rho_*^G \) and \( \rho_*^L \), the liquid pressure averaged on the bubble–liquid interface [12], \( P^* \), is introduced (surface averaged pressure).

As an interfacial momentum transport \( F^* \), we adopt the following model of virtual mass force proposed by Yano et al. [11],

\[
F^* = -\beta_1 \rho_*^G \left( \frac{D_G u_G^*}{Dr^*} - \frac{D_L u_L^*}{Dr^*} \right) - \beta_2 \rho_*^L (u_G^* - u_L^*) \frac{D_G \alpha}{Dr^*} - \beta_3 \alpha (u_G^* - u_L^*) \frac{D_G \rho_*^G}{Dr^*}, \tag{6}
\]

where the coefficients \( \beta_1, \beta_2, \) and \( \beta_3 \) may be set as 1/2 for the spherical bubble, although we proceed without explicitly showing these values to clarify the contribution of each term in the right-hand side of Eq. (6) to the final result. Actually, \( \beta_2 \) and \( \beta_3 \) do not appear in the final result treated in this paper; we note that \( \beta_2 \) affects the nonlinear wave propagation (see, our previous studies treated nonlinear waves in Slow mode [7, 8, 9]). Equation (6) is suggested by the analysis of virtual mass force in a compressible liquid [13, 14].

The dynamics of spherical symmetric oscillations of the representative bubble is determined by the Keller equation [15],

\[
\left( 1 - \frac{1}{c_{LO}^*} \frac{D_G R^*}{Dr^*} \right) R^* \frac{D_G R^*}{Dr^*} + \frac{3}{2} \left( 1 - \frac{1}{c_{LO}^*} \frac{D_G R^*}{Dr^*} \right) \left( \frac{D_G R^*}{Dr^*} \right)^2 = \left( 1 + \frac{1}{c_{LO}^*} \frac{D_G R^*}{Dr^*} \right) P^* + \frac{R^*}{\rho_*^G \rho_*^L \delta_{LO}^*} \frac{D_G}{Dr^*} (p_{LO}^* + P^*), \tag{7}
\]

where \( R^* \) is the radius of the representative bubble, \( p_{LO}^* \) is the liquid density in the initial unperturbed state, and the definitions of operators \( D_G/Dr^* \) and \( D_L/Dr^* \) are

\[
\frac{D_G}{Dr^*} \equiv \frac{\partial}{\partial t^*} + u_G^* \frac{\partial}{\partial x^*}, \quad \frac{D_L}{Dr^*} \equiv \frac{\partial}{\partial t^*} + u_G^* \frac{\partial}{\partial x^*}. \tag{8}
\]

The second term in the right-hand side of Eq. (7) embodies a damping effect on the bubble oscillation and it is mainly responsible for the wave attenuation due to the acoustic radiation from oscillating bubbles.

The set of Eqs. (2)–(7) is closed by the following equations: (i) the Tait equation of state for liquid,

\[
\frac{p_L^*}{\rho_*^G} = \frac{p_{LO}^*}{\rho_*^G} \left( \frac{\rho_{LO}^*}{\rho_*^G} \right)^{n-1}, \tag{9}
\]

where \( n \) is the material constant; e.g., \( n = 7.15 \) for water, (ii) the polytropic equation of state for gas,

\[
\frac{p_G^*}{\rho_*^G} = \left( \frac{\rho_{G0}^*}{\rho_*^G} \right)^{\gamma}, \tag{10}
\]

where \( \gamma \) is the polytropic exponent, (iii) the conservation equation of mass inside the bubble,

\[
\frac{\rho_G^*}{\rho_{G0}^*} = \left( \frac{R_*^i}{R_*^o} \right)^3, \tag{11}
\]

(iv) the balance of normal stresses across the bubble–liquid interface,

\[
p_G^* - (p_L^* + P^*) = \frac{2\sigma^*}{R^*}, \tag{12}
\]

where \( \sigma^* \) is the surface tension. The physical quantities in the initial uniform state at rest are signified by the subscript 0, and they are all constants.

Perturbation expansions

The independent variables, i.e., temporal and spatial coordinates, are firstly nondimensionalized as

\[
t = \frac{t^*}{T^*}, \quad x = \frac{x^*}{L^*}, \tag{13}
\]

where \( T^* \) and \( L^* \) are, respectively, typical periods in time and space of the wave concerned.

The dependent variables are nondimensionalized and expanded in power series of \( \varepsilon \) (a nondimensional wave amplitude

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which is sufficiently small compared with unity), as

\[
\alpha / \alpha_0 = 1 + \varepsilon \alpha_1 + \varepsilon^2 \alpha_2 + \cdots, \quad (14)
\]

\[
u_c^*/U^* = \varepsilon u_{c1} + \varepsilon^2 u_{c2} + \cdots, \quad (15)
\]

\[
a^*/U^* = \varepsilon a_{l1} + \varepsilon^2 a_{l2} + \cdots, \quad (16)
\]

\[
R^*/R_0^* = 1 + \varepsilon R_1 + \varepsilon^2 R_2 + \cdots, \quad (17)
\]

where \( \alpha_0 \) is the initial constant void fraction and \( U^* \) is a typical propagation speed of the wave. The propagation speed \( U^* \), length \( L^* \), and time of waves \( T^* \), are related by \( L^* \equiv U^* T^* \); they are determined in the following section. Although the initial void fraction \( \alpha_0 \) in Eq. (14) should be small compared with unity by the assumptions listed in the second paragraph in preceding subsection, it is treated as a quantity of the order of unity because the asymptotic behavior with respect to the small amplitude \( \varepsilon \) is considered.

Furthermore, the expansion of the liquid density \( \rho_l^* \), in powers of \( \varepsilon \) is assumed as

\[
\rho_l^*/\rho_{l0}^* - 1 = \varepsilon^{\kappa} \rho_{l1} + \varepsilon^{\kappa+1} \rho_{l2} + \cdots, \quad (18)
\]

where \( \kappa \) is an integer number, which is determined as \( \kappa = 1 \) for Band A in Fast mode, as shown in the following section. Substitution of Eq. (18) into Eq. (9) provides the expansion of the liquid pressure \( p_L^* \).

\[
p_L = p_{l0}^* = \frac{p_L^*}{\rho_{l0}^* U^*} = \frac{p_{l0}^*}{\rho_{l0}^* U^*} + \varepsilon^{\kappa-2a} \rho_{l1} + \varepsilon^{\kappa-2a+1} \rho_{l2} + \cdots. \quad (19)
\]

Here, we have introduced \( V \varepsilon^{ad} \) as a measure of the ratio of \( U^* \) and \( c_{l0}^* \) as

\[
\frac{U^*}{c_{l0}^*} = O(\varepsilon^{ad}) \equiv V \varepsilon^{ad} \quad (20)
\]

where a nondimensional parameter \( V \equiv O(1) \) and a real number \( a \) are to be determined. It should be emphasized that \( V \) implies the magnitude of liquid compressibility in the sense that \( V \to 0 \) corresponds to \( c_{l0}^* \to \infty \).

In the pressure waves (or acoustic waves), the perturbation of the liquid pressure should begin with the term of \( O(\varepsilon) \) in Eq. (19), and hence we have

\[
\kappa - 2a = 1, \quad (21)
\]

because the perturbation of the liquid pressure should begin with the term of \( O(\varepsilon) \) in Eq. (19) for the pressure waves concerned.

Equation (19) can then be rewritten into

\[
p_L = p_{l0} + \varepsilon p_{l1} + \varepsilon^2 p_{l2} + \cdots. \quad (22)
\]

The remaining variables, \( p_{g0}^*, \rho_{g0}^* \), and \( p^* \), can also be nondimensionalized and expanded in powers \( \varepsilon \), and their expansion coefficients can be written in terms of \( R_i \) and \( p_{li} (i = 1, 2, \ldots) \) from Eqs. (10)–(12).

The nondimensional pressures for the gas and liquid phases in the unperturbed state \( p_{g0} \) and \( p_{l0} \) are introduced as

\[
p_{g0}^* = \frac{p_{g0}}{\rho_{g0}^* U^*} = O(1), \quad p_{l0}^* = \frac{p_{l0}}{\rho_{l0}^* U^*} = O(1), \quad (23)
\]

respectively. The ratio of initial densities of the gas and liquid phases is assumed to be small as

\[
\frac{\rho_{g0}^*}{\rho_{l0}^*} \equiv O(\varepsilon), \quad (24)
\]

and hence the density ratio does not affect the final result of the present paper. The natural angular frequency of linear spherical symmetric oscillations of a single bubble is also an important parameter, which is given by

\[
\omega_B^* = \sqrt{\frac{3 \gamma (p_{l0}^* + 2 \sigma^*/R_0^*) - 2 \sigma^*/R_0^*}{p_{l0}^* R_0^*}}, \quad (25)
\]

**DERIVATION**

**Parameter scaling**

A set of scaling relations of physical parameters for plane progressive waves in uniform bubbly liquids, including Eq. (20) above, may be written as the following general expression:

\[
\frac{U^*}{c_{l0}^*} = Ve^{ad}, \quad \frac{R_0^*}{L^*} = \Delta e^b, \quad \frac{\omega^*}{\omega_B} = \Omega e^c, \quad (26)
\]

where \( a, b, \) and \( c \) are real numbers; and \( V, \Delta, \) and \( \Omega \) are constants of \( O(1) \).

Waves treated here, i.e., Band A in Fast mode in Fig. 3, are high frequency short waves, and its phase velocity is close to \( c_{l0}^* \). Furthermore, \( b = c \) should be required for a band in the case that the phase velocity is extremely close to the group velocity, as in the case of the KdVB equation for Slow mode (see Fig. 2). As a
result, we now choose the real numbers as \( a = b = c = 0 \), that is,

\[
\frac{U^*}{c_{10}} = V = O(1), \quad (27a)
\]

\[
\frac{\rho_0^*}{L^*} = \Delta = O(1), \quad (27b)
\]

\[
\frac{\omega^*}{\omega_0^*} = \Omega = O(1). \quad (27c)
\]

The choice \( \alpha = 1 \) yields \( \kappa = 1 \) through Eq. (21). All the perturbation expansions therefore begin with the order of \( \epsilon \) including the liquid density (18).

**Linearization**

By the use of the perturbation expansions and nondimensionalizations (14)–(25) with parameter scaling (27), the conservation equations (28)–(31) and Keller equation (32) are, respectively, reduced to

\[
\frac{\partial \rho_{1}}{\partial t} - 3 \frac{\partial R_{1}}{\partial t} + \frac{\partial u_{G1}}{\partial x} = 0, \quad (28)
\]

\[(1 - \alpha_{0}) \frac{\partial p_{1}}{\partial t} - \frac{\partial u_{G1}}{\partial t} + (1 - \alpha_{0}) \frac{\partial u_{1}}{\partial x} = 0, \quad (29)\]

\[
\frac{\beta_{1}}{L^*} \frac{\partial u_{G1}}{\partial t} - \frac{\partial p_{1}}{\partial t} - 3 \frac{\gamma_{p0}^{2}}{\partial x} = 0, \quad (30)\]

\[
(1 - \alpha_{0} + \beta_{1} \alpha_{0}) \frac{\partial u_{1}}{\partial t} - \beta_{1} \alpha_{0} \frac{\partial u_{G1}}{\partial t} + (1 - \alpha_{0}) \frac{\partial p_{1}}{\partial x} = 0, \quad (31)\]

\[
\rho_{1} = \frac{\Delta^2}{\Omega^2} R_{1} - \frac{V^2 \Delta^2}{\Omega^2} \frac{\partial R_{1}}{\partial t} - \Delta^2 \frac{\partial^2 R_{1}}{\partial t^2}. \quad (32)\]

We remark the relation between \( \rho_{1} \) and \( p_{1} \),

\[
\rho_{1} = V^2 p_{1}. \quad (33)\]

Let us note important points in the above set (28)–(33):

(i) The linearized mass-conservation equation in liquid phase (29) includes the time-derivative of \( p_{1} \) [or \( p_{1} \) through Eq. (33)]. We emphasize that, in the derivation of the KdVB and NLS equations for Slow mode [7], the time-derivatives of liquid density \( \rho_{1} \) have appeared not in leading order of approximation but in higher order of approximation (see, detailed derivation procedure, i.e., the equations (35) for KdVB and (61) for NLS in Kanagawa et al. [7]). This reflects that the variation of liquid density in Fast mode is extremely large compared with that in Slow mode.

(ii) The linearized mass- and momentum- conservation equations (28)–(31) are the same as those in the case for Slow mode [7] without only the above point (see, sets (34)–(37) for KdVB and (60)–(63) for NLS in Kanagawa et al. [7]).

(iii) Linearized Keller’s equation (32) includes attenuation term, i.e., first-order time-derivative signified by the coefficient \( V \), which expresses strong attenuation due to acoustic radiation from oscillating bubbles owing to liquid compressibility.

**Resultant single equation**

Let us combine the conservation equations (28)–(31) into the single equation:

\[
\frac{\partial^2 R_{1}}{\partial t^2} - \frac{(1 - \alpha_{0} + \beta_{1}) \gamma p_{0} \partial^2 R_{1}}{\beta_{1} (1 - \alpha_{0}) \partial x^2} + \frac{1}{3 \alpha_{0}} \frac{\partial^2 p_{1}}{\partial x^2} - \frac{V^2 (1 - \alpha_{0}) \partial^2 p_{1}}{3 \alpha_{0}^2} = 0, \quad (34)\]

where the fourth term of the left-hand side is owing to liquid compressibility signified by \( V \), which disappears in the incompressible liquid limit \( V \to 0 \).

Introduction of the relation between \( p_{1} \) and \( R_{1} \) (32) into Eq. (34), we have a linear partial differential equation with constant coefficients for an unknown \( R_{1} \) as:

\[
A_{1}(V) \frac{\partial^4 f}{\partial t^4} + A_{2} \frac{\partial^2 f}{\partial t^2 \partial x^2} + B_{1}(V) \frac{\partial^3 f}{\partial t^3} + B_{2}(V) \frac{\partial^3 f}{\partial x \partial t^2} + C_{1}(V) \frac{\partial^2 f}{\partial t^2} + C_{2} \frac{\partial^2 f}{\partial x^2} = 0, \quad (35)\]

where the constant coefficients are given by

\[
A_{1}(V) = \frac{V^2 \Delta^2 (1 - \alpha_{0})}{3 \alpha_{0}} \geq 0, \quad A_{2} = \frac{\Delta^2}{3 \alpha_{0}} \leq 0, \quad B_{1}(V) = \frac{V^3 \Delta^2 (1 - \alpha_{0})}{3 \alpha_{0} \Omega^2} \geq 0, \quad B_{2}(V) = - \frac{V^2 \Delta^3}{3 \alpha_{0} \Omega^2} \leq 0, \quad C_{1}(V) = 1 + \frac{V^2 \Delta^2 (1 - \alpha_{0})}{3 \alpha_{0} \Omega^2} \geq 0, \quad C_{2} = - \frac{(1 - \alpha_{0} + \beta_{1}) \gamma p_{0} \partial^2 p_{1}}{\beta_{1} (1 - \alpha_{0})} \frac{\Delta^2}{3 \alpha_{0} \Omega^2} \leq 0. \quad (36)\]

Here, the dependence of \( V \) on coefficients are emphasized in brackets. We note that all the coefficients (36) are constants, and Eq. (35) is not a differential equation with variable constants. In the limit of \( V \to 0 \), \( A_{1}, B_{1}, B_{2}, \) clearly go to zero, and \( C_{1} \to 1 \).

All the coefficients are composed of the initial void fraction \( \alpha_{0} \), \( \Delta = \rho_{0}^* / L^* \), and so on. Dependent variables \( f \) can be chosen as other first-order perturbations of physical variables (e.g., \( \alpha_{0}, u_{G1}, u_{1}, \) and \( p_{1} \)), by using the equations (45) and (69) in Kanagawa et al. [7].

The coefficients in fourth-order derivatives, \( A_{1} \) and \( A_{2} \), denote the dispersion effects; and those in third-order derivatives, \( B_{1} \) and \( B_{2} \), denote the dissipation effects, as can be seen the existence of the liquid compressibility \( V \). Equation (35) governs the linear propagation of waves in Band A of Fast mode, i.e., the very high frequency short wave. In the wave propagation process, the dispersion effect and dissipation effect compete with each other; the former is induced by bubble oscillations and the latter is owing to both the acoustic radiation from oscillating bubbles and the liquid viscosity.

Taking incompressible liquid limit, Eq. (35) readily reduces...
to the following dispersive wave equation,
\[
\frac{\Delta^2}{3\alpha_0 \partial^2 \partial x^2} \frac{\partial^4 f}{\partial t^4} + \frac{\partial^2 f}{\partial t^2} - \frac{\partial^2 f}{\partial t^2} + \left[ \frac{(1 - \alpha_0 + \beta_1)\gamma_{pG}}{\beta_1 (1 - \alpha_0)} + \frac{\Delta^2}{3\alpha_0 \Delta^2} \right] \frac{\partial^2 f}{\partial x^2} = 0,
\] (37)
where the dissipation due to liquid compressibility disappears.

CONCLUSIONS

On the basis of parameter scaling for short waves in very high frequency (in Band A of Fast mode),
\begin{align*}
U^*/C_{10}^* &= O(1), & R_0^*/L_0^* &= O(1), & \omega^*/\omega_{B}^* &= O(1),
\end{align*}
we have derived a linear equation for wave propagation with dispersion and dissipation effects:
\[
A_1(V) \frac{\partial^4 f}{\partial t^4} + A_2 \frac{\partial^4 f}{\partial t^2 \partial x^2} + B_1(V) \frac{\partial^3 f}{\partial t^3} + B_2(V) \frac{\partial^3 f}{\partial t \partial x^2} + C_1(V) \frac{\partial^2 f}{\partial t^2} + C_2 \frac{\partial^2 f}{\partial x^2} = 0.
\]

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